(a) Prove that $M'$ is also a projective plane.

IA1: There is a unique line thru every two points in $M$.

In $M$, this means: Every two lines intersect at a unique point in $M$.

That every two lines meet is the elliptic parallel property in $M$. That the intersection is unique follows from Prop 2.1.

IA2': Every line has at least three points in $M$.

In $M$, this means: There are three distinct lines thru every point in $M$.

This is proved as follows. Given a point $P$.

By Prop 2.4, $\exists$ a line $\ell$ not thru $P$.

By IA2' from $M$, $\ell$ has three points $A, B, C$ on it.

Therefore, there are at least three lines $\ell, \ell_B, \ell_C$ thru $P$.

IA3: There are three non-collinear points in $M'$. 

In M, this means: there are three non-concurrent lines.

This is Prop 2.2.

Elliptic Parallel Property: Every two lines intersect at a point in M.

In M, this means: There is a line thru every two points in M. This is IA1.

(b) Let \( A \in \ell_1, A' \in \ell_2 \) be points that are not the intersection C.

Let \( p \) be another point on \( \overrightarrow{AA'} \).

We then establish a 1-1 correspondence between points on \( \ell_1 \) & points on \( \ell_2 \) as follows: Let \( B \) be a point on \( \ell_1 \).

Let \( B' \) be the intersection of \( pB \) & \( \ell_2 \).

Points on \( \ell_1 \) \( \leftrightarrow \) points on \( \ell_2 \)

\( B' \leftrightarrow B \)

Therefore, the number of pts on \( \ell_1 \) = the number of pts on \( \ell_2 \).
Yes, it is a model for incidence geometry,

IA1: For every two points on the punctured sphere, there is a great circle thru P, Q, and N. That is the intersection of the plane thru PQN with the sphere.

IA2: Every great circle in $\mathbb{R}^3$ has infinitely many points.

IA3: There are more than one great circle thru N.

This model satisfies the Euclidean parallel postulate.

(Notice that "N" is removed from the "lines".)
(a) Choose a line $l_1$, not thru $P$.

Each line thru $P$ intersect $l$ at a unique point. Therefore

$\text{# of lines thru } P = \text{# of points on } P = n+1$

(b) As in Part (a), each point in $M$ must be on a line thru $P$. There are $n+1$ such lines, each has $n$ points (excluding $P$). Therefore

$\text{Total number of points}$

$n \cdot (n+1) + 1 = n^2 + n + 1$

for $P$

(c) Given a line $l_2$. Each other line must meet $l_2$ at a point.

There are $n+1$ points on $l_2$.

Each has $n$ lines thru it (excluding $l_2$).

Therefore,

$\text{Total # of lines} = n(n+1) + 1 = n^2 + n + 1$