Exercise I

Remarks: In all the algorithms, always explain their correctness and analyze their complexity. The complexity should be as small as possible. A correct algorithm with large complexity, may not get full credit.

Solve the following 5 questions.

**Question 1:** Let $A$ and $B$ be two sorted arrays of $n$ elements each (thus $A + B$ has $2 \cdot n$ elements in total). Give an algorithm that finds the median of $A \cup B$.

**Question 2:** Given a sorted array $A$ of $n$ numbers and an additional number $x$, give an algorithm that checks if there is $i$ so that $(A[i])^2 = x$.

**Question 3:** Given an array $A$ of size $n - 2$ that contains all the numbers $1, 2, 3, \ldots, n$ except for two. Give an algorithm to find the two missing numbers. Use a constant extra space (namely apart from the array, only a constant number of variables).

**Question 4:** Given an array $A$ with maximum $M$ and minimum $m$.

1. Show that there is a pair of numbers $A[i], A[i + 1]$ so that $|A[i] - A[i + 1]| \leq (M - m)/(n - 1)$

   **Hint:** "Average"

2. Give an algorithm that finds such numbers

**Question 5:** We are given a collection of $n$ arrays, $A_1, A_2, \ldots, A_n$ each with $n$ sorted numbers (every array is sorted). Give an algorithm to get from them one big sorted array of $n^2$ elements.

**Important remark:** You are not allowed to use Priority Queues. There is a way to get a reasonable running time without priority queues.