B:1

Solution to Quiz / H.W. # 2

§7.4: 14; §7.5: 11; §7.7: 42, 44; §7.8: 54; §8.1: 45; §8.2: 23, 64

§7.4 #14 Let P = intensity, t = distance traveled

$$P = P_0 e^{-2t}$$

$$\cancel{P_0} e^{-2t} = \frac{1}{3} \cancel{P_0} \Rightarrow -2t = \ln\left(\frac{1}{3}\right), \quad t = \frac{\ln 3}{2}$$

$$\approx 0.55 \text{ ft}$$

§7.5 #11

The Present value of the profit is (in millions)

$$0.5 e^{-0.06} + 0.5 e^{-0.06\times 2} + 0.5 e^{-0.06\times 3} + 0.5 e^{-0.06\times 4} + 0.5 e^{-0.06\times 5}$$

$$\approx 2.095.$$

Hence the investment is worthwhile.

§7.7 #42

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} \left(\frac{0}{0}\right)$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = \frac{-0}{-1} = 0$$

#44 Let $y = (1 + \ln x)^{\frac{1}{x-1}}$. Then

$$\lim_{x \to 1} \ln y = \lim_{x \to 1} \frac{\ln(1 + \ln x)}{x - 1} \left(\frac{0}{0}\right) = \lim_{x \to 1} \frac{\frac{1}{1+\ln x} \cdot \left(\frac{1}{x}\right)}{1}$$

$$= \frac{1}{(1+0)} \cdot \frac{1}{1} = 1$$

Hence $\lim_{x \to 1} y = e^1 = e.$

(32)

§7.8 #54

$$g(x) = x - \tan^{-1} x$$

$$g'(x) = 1 - \frac{1}{1+x^2} = \frac{x^2}{1+x^2}$$

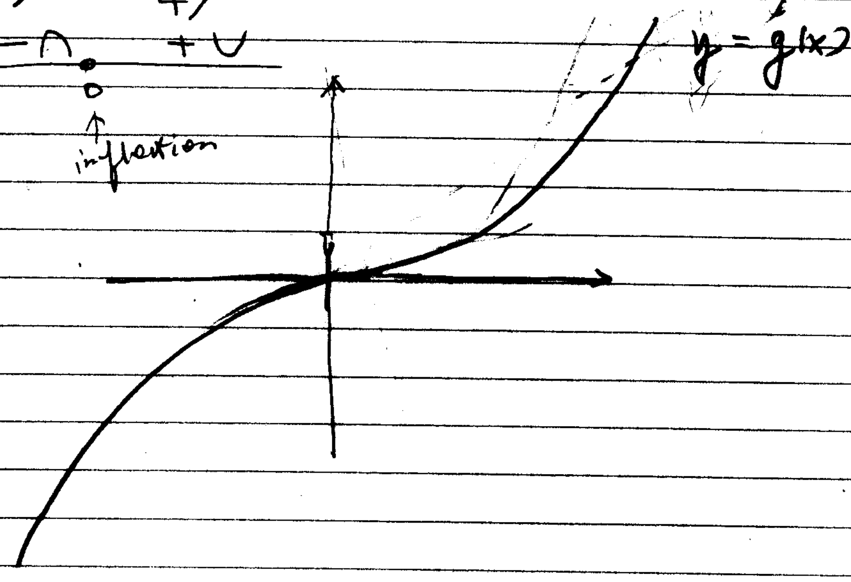
$$g''(x) = \frac{2x(1+x^2) - x^2 \cdot 2x}{(1+x^2)^2} = \frac{2x}{(1+x^2)^2}$$

g': $+ \nearrow$ $\quad + \nearrow$

g'': $- \cap$ $\quad + \cup$

0

↑ inflection



§8.1 #45

$$\text{Error}(S_N) \leq \frac{K_4 (b-a)^5}{180 N^4}$$

$f(x) = x^{\frac{3}{2}}$, $\quad f'(x) = \frac{3}{2} x^{\frac{1}{2}}$, $\quad f''(x) = \frac{3}{4} x^{-\frac{1}{2}}$

$f'''(x) = -\frac{3}{8} x^{-\frac{3}{2}}$, $\quad f^{(4)}(x) = -\frac{9}{16} x^{-\frac{5}{2}}$.

Hence $\quad |f^{(4)}(x)| \leq \frac{9}{16} = K_4 \quad$ on $1 \leq x \leq 7$

$$\frac{\frac{9}{16} \cdot (7-1)^5}{180 N^4} \leq 10^{-9} \Rightarrow N^4 \geq \frac{1 \cdot 9}{180 \cdot 16} \cdot 6^5 \cdot 10^9$$

$$N \geq \left(\frac{9}{180 \cdot 16} \cdot 6^5 \cdot 10^9\right)^{\frac{1}{4}} \approx 394.82.$$

Take $N = 396$. (N must be even for Simpson's Rule).

(B3)

§8.2 #23

$\int (\ln x)^2 dx$ $\quad u = (\ln x)^2, \quad dv = dx$

$du = 2(\ln x)\cdot\frac{1}{x}\,dx; \quad v = x$

$= x(\ln x)^2 - \int \cancel{x}\cdot(2\ln x)\cdot\cancel{\frac{1}{x}}\,dx$

$= x(\ln x)^2 - 2\int \ln x\,dx$

$= x(\ln x)^2 - 2[(x\ln x) - \int x\,d\ln x]$

$= x(\ln x)^2 - 2(x\ln x - \int x\cdot\frac{1}{x}\,dx) = x(\ln x)^2 - 2x\ln x + 2x + C$

#64 $\quad \int \frac{(\ln x)^2}{x^2}\,dx$

Let $u = \ln x$, then $du = \frac{dx}{x}$ & $x = e^u$

Hence $\int \frac{(\ln x)^2}{x}\cdot\frac{dx}{x} = \int \frac{u^2}{e^u}\,du = \int u^2 e^{-u}\,du$

$= -\int u^2\,de^{-u} = -u^2 e^{-u} + \int 2u e^{-u}\,du$

$= -u^2 e^{-u} - 2\int u\,de^{-u}$

$= -u^2 e^{-u} - 2u e^{-u} + 2\int e^{-u}\,du$

$= -u^2 e^{-u} - 2u e^{-u} - 2e^{-u} + C$

$= -e^{-u}(u^2 + 2u + 2)$

$= -e^{-\ln x}((\ln x)^2 + 2(\ln x) + 2)$

$= -\frac{1}{x}((\ln x)^2 + 2(\ln x) + 2) + C$