Math 122—Quiz1, Feb. 04, 2009

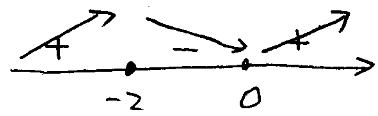
Your Name: Solutions

(1) Find the critical points and determine whether they are local minima, maxima, or neither: $f(x) = x^2 e^x$.

$$f'(x) = 2xe^x + x^2 e^x = (2x + x^2) e^x = x(2+x)e^x$$

$$f'(x) = 0 \Rightarrow x = 0, \quad x = -2$$

(5 pt)



$x = -2$ local max; $y = 4e^{-2}$

$x = 0$ local min; $y = 0$

(5 pt)

(2) Calculate $g(b)$ and $g'(b)$, where g is the inverse of $f(x)$: $f(x) = x + \cos x$, $b = 1$.

$g(b) = a \Rightarrow f(a) = b = 1, \quad a + \cos a = 1$

$\Rightarrow a = 0$

(4 pts)

Hence $g(1) = 0$.

$$g'(b) = \frac{1}{f'(a)} = \frac{1}{1 - \sin a} = \frac{1}{1 - \sin 0} = \frac{1}{1} = 1.$$

(6 pts)

(3) Find derivative using logarithmic differentiation:

$(a) y = x^{e^x}$,

$\ln y = \ln x^{e^x} = e^x \ln x$

$\frac{y'}{y} = e^x \ln x + e^x \cdot \frac{1}{x}$

$\Rightarrow y' = y\left(e^x \ln x + \frac{e^x}{x}\right)$

$= x^{e^x} \cdot e^x \left(\ln x + \frac{1}{x}\right)$

5 pts

$(b) y = \sqrt{\frac{x(x+2)}{(2x+1)(2x+2)}}$

$\ln y = \frac{1}{2}\left(\ln x + \ln(x+2) - \ln(2x+1) - \ln(2x+2)\right)$

$\frac{y'}{y} = \frac{1}{2}\left(\frac{1}{x} + \frac{1}{x+2} - \frac{2}{2x+1} - \frac{2}{2x+2}\right)$

$\Rightarrow y' = \frac{1}{2}\sqrt{\frac{x(x+2)}{(2x+1)(2x+2)}}\left(\frac{1}{x} + \frac{1}{x+2} - \frac{2}{2x+1} - \frac{2}{2x+2}\right)$

5 pts

(4) Evaluate the integral:

$$\int \frac{3\ln x + 4}{x}\, dx$$

$= 3\int \frac{\ln x}{x}\, dx + 4\int \frac{1}{x}\, dx$ ← 4 pts

$\overset{u=\ln x}{=} 3\int u\, du + 4 \ln|x| +$

$= \frac{3}{2}u^2 + 4\ln|x|$

$= \frac{3}{2}(\ln x)^2 + 4 \ln|x| + C$

3 pts 3 pts