Math 113—Test I

March 9, 2006

Your Name: Solutions

Solve exactly eight of the following ten problems. If you solve more then 10 problems, you must indicate which ones are to be graded. Each problem worths 10 points.

(1) Simplify.

5 (a) $\frac{1}{x-y} - \frac{x}{x^2-y^2} + \frac{1}{2x+2y}$

$L.C.D = 2(x-y)(x+y)$

$= \frac{2(x+y) - 2x + x - y}{2(x-y)(x+y)}$ 3pts

$= \frac{\cancel{x+y}}{2(x-y)\cancel{(x+y)}}$

$= \frac{1}{2(x-y)}$

5 (b) $\dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

$= \dfrac{\frac{x^2-(x+h)^2}{(x+h)^2 x^2}}{h}$ 2pts

$= \frac{-2xh - h^2}{h(x+h)^2 x^2}$

$= \frac{-2x-h}{(x+h)^2 x^2}$

(2) Simplify, using no negative exponents in the final answers.

5 (a) $\left(\frac{x}{y}\right)^{-2}\left(\frac{2y}{z}\right)^{-1}\frac{(2x)^3}{z^3}$

$= \left(\frac{y}{x}\right)^2 \cdot \left(\frac{z}{2y}\right) \cdot \frac{8x^3}{z^3}$ 2 pts

$= \frac{y^2 z \cdot 8x^3}{x^2 \cdot 2y \cdot z^3}$

$= \frac{4yx}{z^2}$

5 (b) $\frac{(u\sqrt{v\sqrt{w}})(v\sqrt{w\sqrt{u}})}{w\sqrt{uv\sqrt{uw}}}$

$= \frac{u \cdot \cancel{v^{\frac{1}{2}}} \cdot \cancel{w^{\frac{1}{4}}} \cdot v \cdot w^{\frac{1}{2}} \cancel{u^{\frac{1}{4}}}}{w \; u^{\frac{1}{2}} v^{\frac{1}{2}} \cdot \cancel{u^{\frac{1}{4}}} \cdot \cancel{w^{\frac{1}{4}}}}$ 2pts

$= \frac{u^{\frac{1}{2}} \cdot v}{w^{\frac{1}{2}}} = \frac{v\sqrt{u}}{\sqrt{w}}$

(3) Find the center and radius of the circle: $2x^2 + 2y^2 - 4x + 6y = 0$.

$$x^2+y^2-2x+3y=0$$

$$(x^2-2x+1)+(y^2+3y+\tfrac{9}{4}) = 1+\tfrac{9}{4}$$

$$(x-1)^2+(y+\tfrac{3}{2})^2 = \tfrac{13}{4}$$ 5 pts

Center $= (1, -\frac{3}{2})$, 3pt Radius $= \frac{\sqrt{13}}{2}$ 2pt

(4) Find an equation for the line that has x-intercept -2 and is perpendicular to the line through the points $(4, 1)$ and $(1, 4)$.

Slope of the line thru $(4,1)$ & $(1,4)$ $= \dfrac{4-1}{1-4} = -1$

Slope of the perpendicular line $= -\frac{1}{-1} = 1$

5 pts

point $(-2, 0)$

$y - 0 = 1(x+2)$ 5 pts $y = x+2$

(5) Solve the equations.

(a) $\dfrac{2}{x^2-1} - \dfrac{x}{x-1} = \dfrac{1}{x+1}$

$(x^2-1)\left(\dfrac{2}{x^2-1} - \dfrac{x}{x-1}\right) = (x^2-1)\cdot\dfrac{1}{x+1}$

$2 - x(x+1) = x-1$

2pt

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3, \quad x=1$ 2pt

Check $x=-3$: LHS $= \frac{2}{8} - \frac{-3}{-4} = -\frac{1}{2}$

RHS $= \frac{1}{-2}$ Yes!

2pt

$x = 1$, NO, Denominator $=0$

(b) $\sqrt{x} - \sqrt{3x-3} = -1$

$\sqrt{x}+1 = \sqrt{3x-3}$

$x + 2\sqrt{x} + 1 = 3x-3$

1 pt

$2\sqrt{x} = 2x-4$

$\sqrt{x} = x-2$

$x = x^2-4x+4$

$x^2-5x+4=0$

$(x-1)(x-4)=0$ 2pt

$x=1, \quad x=4$

Check: $x=1$, LHS $= 1-0=1$ 2pt

RHS $=-1$ NO!

$x=4$: LHS $= 2-3=-1=$ RHS

Yes!

(6) Solve the quadratic equations.

(a) $3x^2 + 5x - 1 = 0$

$$x = \frac{-5 \pm \sqrt{25+12}}{6}$$

$$= \frac{-5 \pm \sqrt{37}}{6}$$

5 pts

(b) $x^2 + 3x + 3 = 0$

$D = 9 - 3 \cdot 4 = -3 < 0$

no real solutions.

5 pts

(7) Factor completely

(a) $4x^2 - y^2 + 4y - 4$

$= (2x)^2 - (y^2 - 4y + 4)$

$= (2x)^2 - (y-2)^2$ 3 pts

$= (2x - (y-2))(2x + (y-2))$

$= (2x - y + 2)(2x + y - 2)$

2 pt

(b) $y^4 z - z^4 y$

$= zy(y^3 - z^3)$ 3 pt

$= zy(y-z)(y^2 + yz + z^2)$

2 pt

(8) Assume $f(x) = x^2 - 2$ and $g(z) = x + 2$. Solve for x such that $2f(x) - g(x) = g(f(x))$.

$2(x^2 - 2) - (x+2) = (x^2 - 2) + 2$ 5 pt

$2x^2 - 4 - x - 2 = x^2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, \ x = -2$ 5 pt

(9) Solve and express the solution set with interval notation:

(a) $(x-3)x \le (x-3)x^2$ (b) $|\frac{1}{2}x - 1| > \frac{3}{2}$

$(x-3)x^2 - (x-3)x \ge 0$

$(x-3)(x^2-x) \ge 0$

$x(x-3)(x-1) \ge 0$ 2pts

		0	1	3	
x	$-$	$+$	$+$	$+$	
$x-3$	$-$	$-$	$-$	$+$	
$x-1$	$-$	$-$	$+$	$+$	2pt
	$-$	$+$	$-$	$+$	

Solution $[0, 1]$, $[3, \infty)$ 1pt

$\frac{1}{2}x - 1 > \frac{3}{2}$ or $\frac{1}{2}x - 1 < -\frac{3}{2}$

$\frac{1}{2}x > \frac{5}{2}$ or $\frac{1}{2}x < -\frac{1}{2}$

$x > 5$ or $x < -1$

2pt 2pt

$(-\infty, -1) \cup (5, \infty)$

7pt

(10) An object is thrown upward from the top of a 64-foot tall building with an initial velocity of 48 feet per second. Find its maximal height and when it will hit the ground. (The height $s(t)$ after t seconds is given by $s = -\frac{1}{2}gt^2 + v_0 t + s_0$ where $g = 32$ ft/sec^2 is the gravitational constant, v_0 the initial velocity, s_0 the initial height.)

$S = -16t^2 + 48t + 64$

$= -16(t^2 - 3t) + 64$

$= -16(t^2 - 3t + \frac{9}{4} - \frac{9}{4}) + 64$

$= -16(t - \frac{3}{2})^2 + 36 + 64 = -16(t - \frac{3}{2})^2 + 100$ 5 pt

max height 100 ft $\frac{3}{2}$ second after. 2pt

$S = 0$ $16(t - \frac{3}{2})^2 = 100$

$(t - \frac{3}{2})^2 = \frac{100}{16} = \frac{25}{4}$

$t - \frac{3}{2} = \pm \frac{5}{2}$

$t = \frac{3}{2} \pm \frac{5}{2}$ $t = 4$ or $\cancel{t=-1}$

Hit the ground 4 second after. 3pt

Math 113—Test II

April 20, 2006

Your Name: Solutions

Solve exactly eight of the following nine problems. If you solve more then eight problems, you must indicate which ones are to be graded. Each problem worths ten points.

(1) Find the inverse $f^{-1}(x)$ of $y = f(x)$ if $f(x) = \dfrac{2x-1}{2x+1}$. Determine the domain and the range of f^{-1}.

$y = \frac{2x-1}{2x+1}$ $\quad (2x+1)y = 2x-1 \quad 2xy + y = 2x - 1$

$2xy - 2x = -1 - y \quad x(2y-2) = -1-y$

$x = \frac{-1-y}{2y-2} \quad y = f^{-1}(x) = \frac{-1-x}{2x-2}$ 6 pts

$Dom(f^{-1}) = \{x \neq 1\}$ 2pts

$Range(f^{-1}) = Dom(f) = \{x \neq -\frac{1}{2}\}$ 2pts

(2) Determine the quotient and remainder when $f(x) = x^5 + 2x^3 - 1$ is divided by $g(x) = x^3 - 1$.

$$
\begin{array}{r}
x^2 + 2 \\
x^3 - 1 \,\big)\, \overline{x^5 + 2x^3 - 1} \\
-)\ x^5 - x^2 \\
\hline
2x^3 + x^2 - 1 \\
-)\ 2x^3 \quad\quad - 2 \\
\hline
x^2 + 1
\end{array}
$$

x^2+2 ← Quotient 2 pt

6 pts

x^2+1 ← Remainder 2 pt

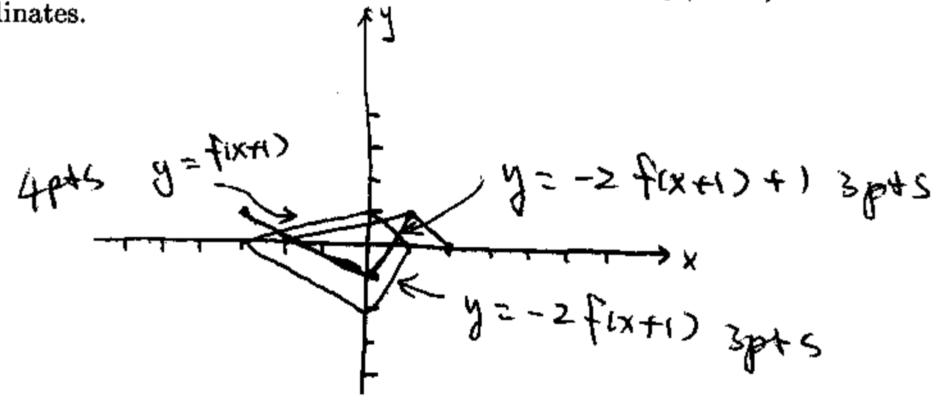
(3) Given that one of the zeros of $x^3 - 5x^2 - 5x + 25$ is 5. Factor the polynomial completely.

$$\begin{array}{r|rrrr} 5 & 1 & -5 & -5 & 25 \\ & & 5 & 0 & -25 \\ \hline & 1 & 0 & -5 & \boxed{0} \end{array} \Bigg\} \text{5 pts}$$

$$x^3-5x^2-5x+25 = (x-5)(x^2-5) \leftarrow \text{2 pts}$$

$$= (x-5)(x-\sqrt{5})(x+\sqrt{5}) \leftarrow \text{3 pts}$$

(4) The following figure gives the graph of $y = f(x)$ with domain $[-2, 2]$. Draw the graph of $y = f(x+1)$, $y = -2f(x+1)$, and $y = -2f(x+1) + 1$ on the same coordinates.



(5) (a) Solve for x using the logarithmic form: $2 \cdot 3^{5x} = 7$.
(b) Solve for x using exponential form: $\log_3(5x) = \sqrt{2}$.

(a) $3^{5x} = \frac{7}{2}$ $\quad 5x = \log_3 \frac{7}{2}$ (3pts) $\quad x = \frac{1}{5}\log_3 \frac{7}{2}$ (2pts) ≈ 0.228

(b) $5x = 3^{\sqrt{2}}$ (3pts) $\quad x = \frac{1}{5} 3^{\sqrt{2}} \approx 0.946$ (2pts)

(6) (a) Express the logarithm in terms of $\log_b x$, $\log_b y$, and $\log_b z$: $\log_b \sqrt{xy^3/z^5}$.
(b) Rewrite the expression $\log_b \sqrt{b} - 3\log_b \sqrt{b}$ as a single logarithm and simplify.

(a) $\frac{1}{2}(\log_b xy^3/z^5) = \frac{1}{2}(\log_b x + 3\log_b y - 5\log_b z)$

$= \frac{1}{2}\log_b x + \frac{3}{2}\log_b y - \frac{5}{2}\log_b z$

5 pts

(b) $\log_b \frac{\sqrt{b}}{(\sqrt{b})^3} = \log_b \frac{1}{b} = \log_b b^{-1} = -1$

5 pts

(7) (a) Solve for x: $9^{2x-1} = 2^{9x+1}$. (You can leave your answer in terms of common logarithms.)
(b) Solve for x: $\log(x^2-4) - \log(x+2) = 3$.

(a) $(2x-1)\log 9 = (9x+1)\log 2$ 3 pts

$2x\log 9 - 9x\log 2 = \log 2 + \log 9$

$x = \frac{\log 2 + \log 9}{2\log 9 - 9\log 2} \approx -1.568$

2 pts

(b) $\log\frac{x^2-4}{x+2} = 3$ 2 pts

$\log(x-2) = 3$

$x - 2 = 10^3 = 1000$

$x = 1002.$ 3 pts

Check: LHS $= \log(1002^2-4) - \log(1002+2) = \log\frac{1004000}{1004}$

$= \log 1000 = 3 =$ RHS

(8) A principal amount of $P = \$1500$ is invested in a saving account that has annual interest rate of 6%. Suppose interest is compounded annually. How many year would it take for the investment to reach \$2500?

$$25\cancel{00} = (1+0.06)^t \cdot 15\cancel{00} = 15(1.06)^t$$

$$5 = 3(1.06)^t \quad 5 \text{ pts}$$

$$\ln 5 = \ln 3 + t \ln 1.06$$

$$t = \frac{\ln 5 - \ln 3}{\ln 1.06} \approx 8.77 \text{ years}$$

5 pts

It takes about 9 years for the investment to reach \$2500.

(9) The half-life of radioactive material strontium-90 is 29 years. Suppose that the initial amount is 2 milligrams.
(a) How much will remain after three half-lives?
(b) How much will remain after 10 years?
(c) When will 1.2 milligrams remain?

(a) $\cancel{2} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25$ milligrams

3 pts There are 0.25 milligrams remaining after 3 half-lives

(b) $Q = 2\left(\frac{1}{2}\right)^{\frac{10}{29}} = 2e^{-\frac{10}{29}\ln 2} \approx 1.5748$

4 pts

$$\left(Q = 2\left(\tfrac{1}{2}\right)^{\frac{t}{29}} = 2e^{-\frac{\ln 2}{29}t} \approx 2e^{-0.0239t}\right)$$

1.5748 mg remains after 10 years

(c) $2e^{-0.0239t} = 1.2$

3 pts

$$\ln 2 - 0.0239t = \ln 1.2$$

$$t = -\frac{\ln 1.2 - \ln 2}{\frac{\ln 2}{29}} \approx 21.37 \text{ years}$$

After 21.37 years, 1.2 mg remains