Math 113—Quiz I

February 2, 2006

Your Name: Solutions!

Solve all five problems. Each problem worths 10 points.

(1) In 1975, 25 % of the population of Northern American was less than 15 years old, and 42 % of population of Southern American was less than 15 years old. The populations of Northern America and Southern America were approximately 237 million and 324 million, respectively. What percent of the total population of Northern and Southern America was less than 15 years old?

Total population = 237 + 324 = 561 million

Total population under 15 year old = $237 \times 25\% + 324 \times 42\%$

$= 195.33$ million

$$\text{Percentage} = \frac{195.33}{561} = 0.3482 \approx 34.82\%$$

(2) Simplify, using no negative exponents in the final answers. Assume that n is a positive integer.

(a) $\dfrac{x^{-3}y^4}{(-x^2y^2)^{-3}}$

$$= \frac{x^{-3}y^4}{-x^{-6}y^{-6}}$$

$$= -x^{-3+6}y^{4+6}$$

$$= -x^3y^{10}$$

(b) $\left(\dfrac{x}{y}\right)^{-2n}\left(\dfrac{2y}{z}\right)^{-n}\dfrac{(2x)^{3n}}{z^{3n}}$

$$= \frac{x^{-2n}}{y^{-2n}} \cdot \frac{2^{-n}\cdot y^{-n}}{z^{-n}} \cdot \frac{2^{3n}\cdot x^{3n}}{z^{3n}}$$

$$= 2^{3n-n} \cdot x^{-2n+3n} \cdot y^{-n+2n} \cdot z^{n-3n}$$

$$= 2^{2n} \cdot x^n \cdot y^n \cdot z^{-2n}$$

$$= \frac{2^{2n}\cdot x^n \cdot y^n}{z^{2n}} = \frac{4^n x^n y^n}{z^{2n}}$$

(3) Simplify. The letters represent positive real numbers.

(a) $x\sqrt{2x} - \sqrt{18x^3} + x\sqrt{50x}$

$= x\sqrt{2x} - \sqrt{(3x)^2 \cdot 2x} + x\sqrt{25 \cdot 2x}$

$= x\sqrt{2x} - 3x\sqrt{2x} + 5x\sqrt{2x}$

$= (x - 3x + 5x)\sqrt{2x}$

$= 3x\sqrt{2x}$

(b) $\dfrac{(u\sqrt{v\sqrt{w}})(v\sqrt{w\sqrt{u}})}{w\sqrt{vw\sqrt{uw}}}$

$= \dfrac{u \cdot (v \cdot w^{\frac{1}{2}})^{\frac{1}{2}} \cdot v \cdot (wu^{\frac{1}{2}})^{\frac{1}{2}}}{w\,(vw(uw)^{\frac{1}{2}})^{\frac{1}{2}}}$

$= \dfrac{u \cdot v^{\frac{1}{2}} \cdot w^{\frac{1}{4}} \cdot v \cdot u^{\frac{1}{4}} \cdot w^{\frac{1}{4}}}{w \cdot v^{\frac{1}{4}} \cdot w^{\frac{1}{4}} \cdot u^{\frac{1}{4}} \cdot w^{\frac{1}{4}}}$

$= \dfrac{u \cdot v}{w}$

(4) Compute and simplify:

(a) $3u[v + \frac{5}{2}(1 - v)] - v[3u + \frac{1}{2}(1 - u)]$

$= 3u(v + \frac{5}{2} - \frac{5}{2}v) - v(3u + \frac{1}{2} - \frac{1}{2}u)$

$= 3u(\frac{5}{2} - \frac{3}{2}v) - v(\frac{5}{2}u + \frac{1}{2})$

$= \frac{15}{2}u - \frac{9}{2}uv - \frac{5}{2}uv - \frac{1}{2}v$

$= \frac{15}{2}u - 7uv - \frac{1}{2}v$

(b) $(y - 1)(y^3 + y^2 + y + 1)$

$= y^4 + y^3 + y^2 + y - y^3 - y^2 - y - 1$

$= y^4 - 1$

(5) Factor completely

(a) $4u^2 - v^2 + 6v - 9$

$= 4u^2 - (v^2 - 6v + 9)$

$= (2u)^2 - (v - 3)^2$

$= (2u - (v - 3))(2u + (v - 3))$

$= (2u - v + 3)(2u + v - 3)$

(b) $y^{4n}z^n - y^n z^{4n}$

$= y^n z^n (y^{3n} - z^{3n})$

$= y^n z^n ((y^n)^3 - (z^n)^3)$

$= y^n z^n (y^n - z^n)(y^{2n} + y^n z^n + z^n)$

$\hookrightarrow (y - z)(y^{n-1} + y^{n-2}z + \cdots + z^{n-1})$

this step is not required.

Quiz #2: Solutions

§6 #17 $$\frac{(1-s^3)/(1+s)}{(1-s)/(1+s^3)} = \frac{1-s^3}{1+s}\cdot\frac{1+s^3}{1-s}$$

$$= \frac{(1-s)(1+s+s^2)(1+s)(1-s+s^2)}{(1+s)(1-s)} = (1+s+s^2)(1-s+s^2)$$

#50 $$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \frac{\frac{x^2-(x+h)^2}{(x+h)^2x^2}}{h}$$

$$= \frac{x^2-(x^2+2xh+h^2)}{(x+h)^2\cdot x^2\cdot h} = \frac{-h(2x+h)}{(x+h)^2\cdot x^2\cdot h} = -\frac{2x+h}{(x+h)^2\cdot x^2}$$

§7 #34 multiply both sides by $(y+1)(y-1)$:

$$2(y-1)-3(y+1) = -4$$

$$-y-5 = -4 \qquad y = -1$$

Check: when $y=-1$, denominator $=0$, not a solution.

#71 Let x = # of correct answers.

Then # of incorrect unknown $= 25-x$

$$30+4x-(25-x) = 95$$

$$5x = 90 \qquad \boxed{x=18}$$

The student answer 18 questions correctly.

§8 #50 $$D = 3^2-4(1)(-1) = 9+4 = 13$$

$$x = \frac{-3\pm\sqrt{13}}{2} \quad \text{— solutions.}$$

§8 #83 $\frac{S}{P} = (1+r)^2$

$\Rightarrow 1+r = \sqrt{\frac{S}{P}}$ (we take + sign because $1+r \geq 0$)

$\boxed{r = \sqrt{\frac{S}{P}} - 1}$

§9 #9 $\sqrt{w} + 2 = \sqrt{2w+7}$

Squaring both sides:

$w + 4\sqrt{w} + 4 = 2w + 7$

$4\sqrt{w} = w + 3$

Squaring again: $16w = w^2 + 6w + 9$

$w^2 - 10w + 9 = 0$

$(w-9)(w-1) = 0$

$w = 1, \quad w = 9$

check: For $w=1$, LHS $= -2$ RHS $= -2$ Yes!

For $w=9$ LHS $= -2$ RHS $= -2$ Yes!

#25 Let $y = 1 + \frac{1}{x}$

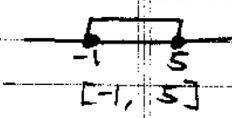
$y^2 + 5y + 4 = 0 \quad (y+4)(y+1) = 0$

$\Rightarrow y = -4, \quad y = -1$

$1 + \frac{1}{x} = -4 \Rightarrow \frac{1}{x} = -5$ $\boxed{x = -\frac{1}{5}}$

$1 + \frac{1}{x} = -1 \Rightarrow \frac{1}{x} = -2$ $\boxed{x = -\frac{1}{2}}$

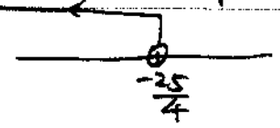
§10 #39 $-\frac{3}{2} \leq \frac{1}{2}x - 1 \leq \frac{3}{2} \Rightarrow -\frac{1}{2} \leq \frac{1}{2}x \leq \frac{5}{2} \Rightarrow -1 \leq x \leq 5$



$[-1, 5]$

#56 $D = 5^2 - 4(-1)p < 0$

$25 + 4p < 0 \Rightarrow 4p < -25 \Rightarrow p < -\frac{25}{4}$



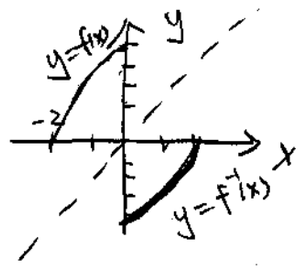
$(-\infty, -\frac{25}{4})$

H1

H.W. #3 §17: 39, 41; §18: 5, 29. §19: 8, 26
§20: 23, 26, §24: 14, 23

§17 #39

(a)



The function has an inverse because it satisfies the horizontal line Test.

(b) $y = -x^2 + 4$ $\quad x^2 = 4 - y$

$x = \pm\sqrt{4-y}$, take $x = -\sqrt{4-y}$ since $x \le 0$.

Thus $y = f^{-1}(x) = -\sqrt{4-x}$

§17 #41

$$y = \frac{2x-1}{x+3} \qquad x \neq -3$$

$(x+3)y = 2x - 1$ $\quad xy - 2x = -3y - 1$

$x(y-2) = -3y - 1$ $\quad x = \dfrac{-3y-1}{y-2}$ $\quad$ Range (f) =

$f^{-1}(x) = -\dfrac{3x+1}{x-2}$ $\quad$ Dom $(f^{-1}) = \{x \mid x \neq 2\}$

Range (f^{-1}) = Dom (f)

$= \{x \mid x \neq -3\}$

§18 #5

$$\begin{array}{r} 3x - 6 \quad \leftarrow \text{Quotient} \\ x^2+2x+1 \;\overline{)\; 3x^3 + 0x^2 - 8x - 4} \\ -)\; 3x^3 + 6x^2 + 3x \\ \hline -6x^2 - 11x - 4 \\ -)\; -6x^2 - 12x - 6 \\ \hline x + 2 \leftarrow \text{Remainder} \end{array}$$

$$\frac{3x^3 - 8x - 4}{x^2+2x+1} = 3x - 6 + \frac{x+2}{x^2+2x+1}$$

(H2)

§1.8 #29

$$\begin{array}{c|ccccc} \frac{1}{3} & 5 & 0 & -2 & 0 & -1 \\ & & \frac{5}{3} & \frac{5}{9} & -\frac{13}{27} & -\frac{13}{81} \\ \hline & 5 & \frac{5}{3} & -\frac{13}{9} & -\frac{13}{27} & \boxed{-\frac{94}{81}} \end{array} = f\left(\tfrac{1}{3}\right)$$

$$\begin{array}{c|ccccc} -0.1 & 5 & 0 & -2 & 0 & -1 \\ & & -0.5 & 0.05 & 0.195 & -0.0195 \\ \hline & 5 & -0.5 & -1.95 & 0.195 & \boxed{-1.0195} \end{array} = f(-0.1)$$

§1.9 #8

$$\begin{array}{c|ccccccc} +0.2 & 5 & -1 & 0 & 0 & 10 & -2 & 1 \\ & & 1 & 0 & 0 & 0 & 2 & 1 \\ \hline & 5 & 0 & 0 & 0 & 10 & 0 & \boxed{1} \end{array} = f(0.2) \neq 0$$

$x-0.2$ is not a factor

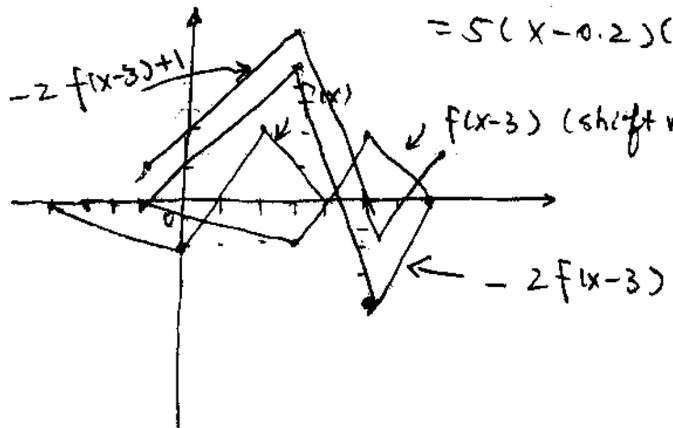
§1.9 #26

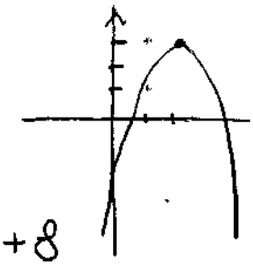
$$\begin{array}{c|cccc} 0.2 & 5 & -1 & -10 & 2 \\ & & 1 & 0 & -2 \\ \hline & 5 & 0 & -10 & \boxed{0} \end{array}$$

$(x-0.2)(5x^2-10) = 5(x-0.2)(x^2-2)$

$= 5(x-0.2)(x-\sqrt{2})(x+\sqrt{2})$

§2.0 #23





§2.0 #26

$f(x) = -2x^2+8x-5 = -2(x^2-4x+4)-5+8$

$= -2(x-2)^2+3$

(H3)

§24 #14 $(h \circ g)(1) = h(g(1)) = h(2) = 3^2 = 4$

(#23) $(b^x + b^{-x})^2 - (b^x - b^{-x})^2 = 4$

$b^{2x} + 2b^x \cdot b^{-x} + b^{-2x} - (b^{2x} - 2b^x \cdot b^{-x} + b^{-2x}) = 4$

$2b^0 + 2b^0 = 4$

$4 = 4$ ✓