Exercise 4

Remarks: All the graphs here are without self loops and parallel edges, and anti-parallel edges. In all the algorithms, always explain their correctness and analyze their complexity. The complexity should be as small as possible. A correct algorithm with large complexity, may not get full credit.

- **Question 1:** Consider an input $G(V, E)$ with costs $c(e) > 0$ for every $e$. Consider a vertex $v$ and the min cost edge $e_v$ touching $v$. Show that there exists a minimum spanning tree that contains $e_v$.

- **Question 2:** Given a graph $G(V, E)$ with positive and pairwise different weights on the edges. Show that the minimum spanning tree is unique.

- **Question 3:** A diameter in a $T(V, E)$, is the length (number of edges) of the longest simple path in the tree.

  1. Show that there exists a vertex that belongs to all the diametres in a tree (recall that a path is a diameter in a tree if its length is the largest possible length between two vertices in the tree).

  2. Give an algorithm that finds such vertex.

- **Question 4:** We are given an undirected graph with costs $c(e)$ on the edges that may be positive or negative. Give an algorithm that finds the min cost connected subgraph of $G$.

- **Question 5:** Run the Prim algorithm on the following graph: All you need to do (as in class) is copy the vertices and the tree edges only. On the edges you write a number between 1 and 7, representing the order by which the edge is added into the solution.
Figure 1: Run Prim on this graph